ANALYSIS OF SWITCHING ACTIVITY IN DSP SIGNALS IN THE PRESENCE OF NOISE

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***Abstract*—Input switching activity is one of the deciding factors for power consumption in digital signal processing components. For accurate power estimation, it is essential to have knowledge about the switching activity in the input signal, including how this activity changes in different environments, e.g., in the presence of noise. The Dual Bit Type (DBT) method aims at characterizing the bit-level switching activity in a signal, using signal statistics. However, the DBT method requires that the correlation coefficient and switching activity for the most significant bit of the signal are available. In this paper we give an expression for direct calculation of the correlation coefficient for the most significant bit in a signal, using the word-level correlation coefficient. Using simulation results we examine the accuracy of the given method to calculate the switching activity and correlation coefficient for the most significant bit. Furthermore, we derive expressions for accurately calculating the variance and word-level correlation coefficient for a correlated signal, when an additional noise of a given variance is added to the signal. This can be used to estimate the bit-level switching activity in a signal in the presence of noise. Finally, based on this we study the impact the additional noise has on the switching activity of the resulting signal.**

***Index Terms*—switching activity, DSP signals, dual bit type, bit-level switching activity**

I. INTRODUCTION

Through the last decade, power consumption in CMOS VLSI circuits has become a major design constraint. In order to design circuits for low power, it is now essential to be able to accurately estimate power consumption at different design stages, at circuit and logic level as well as at architectural level. Since dynamic power consumption in a CMOS VLSI circuit depends on the number of signal transitions at its capacitive nodes, accurate estimation of the bit-level switching activity at the primary inputs is a key requirement in various power estimation techniques. The input switching activity can subsequently be used directly or indirectly to calculate the number of transitions of all nodes in the circuit [1].

The Dual Bit Type method introduced in [2] aims at characterizing the bit-level switching activity in a data word, using the word-level statistics of data, i.e., mean, μ, variance, σ^2, and temporal correlation, ρ. The method is based on the assumption that the binary representation of real world signals can be divided into a few regions, with well defined switching activity for the bits in each region. In [2] the binary representation of a word is divided into three regions: the most significant bit (MSB) region or sign region (S), the uniform white noise (UWN) region, and an intermediate region. The bits in the UWN region, which are defined to last from the least significant bit to a certain breakpoint P_0, exhibit random switching with switching activity equal to $\alpha_{UWN} = 0.5$. Switching activity for the bits in the S-region, which lasts from the most significant bit to another breakpoint P_1, is a constant, while the switching activity in the intermediate region is assumed to decrease in a linear manner from α_{UWN} to α_{MSB}, where α_{UWN} and α_{MSB} are the switching activities in the UWN and S regions, respectively.

Experiments have shown that the method presented in [2] to calculate the regions boundaries, P_0 and P_1, is not accurate for highly correlated signals. The work in [3], which builds on the principles of [2], has however developed expressions which take correlation of data into account when defining the mentioned regions. Accurate estimation of bit-level switching activity, using expressions in [3], requires accurate estimation of ρ_{MSB} and α_{MSB}, where ρ_{MSB} and α_{MSB} are the temporal correlation and the switching activity for the most significant bit, respectively. In [3] the method presented to calculate α_{MSB} and ρ_{MSB} is based on knowledge of a signal generation model. ρ_{MSB} in [2] was acquired using simulation.

An exact method to calculate the bit-level correlation factor ρ_{MSB} was presented in [4]. It is again based on knowledge of a signal generation model. The estimation method calculating α_{MSB} assumes, however, that ρ_{MSB} is equal to the word-level correlation ρ. This is not accurate in many cases. In [5] an accurate analytical expression for calculating α_{MSB} using the word-level correlation ρ is presented. In this paper we use this expression for α_{MSB} to derive an expression for direct calculation of ρ_{MSB}. We also use simulation to examine the accuracy of this method to estimate α_{MSB} and ρ_{MSB}.

Furthermore, signals are often subjected to noise from different sources, e.g., quantization noise from fixed point DPS realization, wired and wireless communication channel noise. This changes the word-level signal statistics, σ^2 and ρ. As mentioned above, the methods

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for calculating the bit-level switching activity require that σ^2 and ρ are available. Motivated by this we derive expressions for accurately calculating σ^2 and ρ when an additional noise of a given variance is added to the signal. This makes accurate estimation of switching activity possible in such conditions. We also discuss how such additional noise impacts the switching activity in the signal.

The organization of this paper is as follows. The data model we use, and the method for calculating the bit-level switching activity, are described in Section II and III, respectively. In Section IV our expression allowing direct calculation of ρ_{MSB} from ρ is derived and using simulation, the accuracy of the given method to estimate α_{MSB} and ρ_{MSB} is examined. In Section V we derive expressions for accurately calculating σ^2 and ρ for a noisy signal $z(n)$, and discuss how this additional noise impacts the switching activity of a signal. Finally our main conclusions are summarized in Section VI.

II. Data model

In a wide spectrum of DSP applications, the input data signal $z(n)$ can be assumed to have a Gaussian distribution with mean μ_z, variance σ_z^2, and lag-l correlation ρ_z [4]. Here $\mu_z = E[z(n)]$, $\sigma_z^2 = E[(z(n)-\mu_z)^2]$, and $\rho_z = \frac{E[z(n)z(n-1)]-\mu_z^2}{\sigma_z^2}$. The signal $z(n)$ could, e.g., be the resulting signal when an original correlated signal $d(n)$, with a given ρ, is subjected to an additive white noise $w(n)$:

$$z(n) = d(n) + w(n). \quad (1)$$

The *lag-1 switching activity* of the *i*th bit of the W-bit signal $z(n)$, α_i, is defined as a probability:

$$\alpha_i = \mathrm{P}(z_i(n) \neq z_i(n-1)) \quad (2)$$

As a particular case, we consider the channel estimator of a wireless receiver. The input to this module is a signal $d(n)$, resulting from a transmitted pilot signal which typically has passed through a Rayleigh fading channel. The resulting signal is a complex valued signal that can be described as two independent Gaussian random processes with zero mean. Assuming *isotropic scattering* for the fading environment, the fading is said to have a Jakes spectrum, and the autocorrelation function for the faded pilot signal is given by:

$$R_d(k) = E[d(n)d(n+k)] = \sigma^2 \cdot J_0(2\pi f_D k T_s) \quad (3)$$

where J_0 is the zeroth-order Bessel function of the first kind, T_s is the sample period, and f_D is the maximum Doppler frequency ($f_D = (\nu/c)\cdot f_c$, where ν is the speed of the mobile terminal, and f_c is the carrier frequency).

In our experiments to follow, we will use a Rayleigh fading simulator, described in [6], to generate correlated data $d(n)$, with varying Doppler rate $D = f_D T_s$. We use sample period $T_s = 25\mu s$, and carrier frequency $f_c = 2GHz$ in our experiments. We emphasis, however, that although we use this simulator to generate the correlated signal $d(n)$, the theoretical methods presented in this paper are not limited to this particular case.

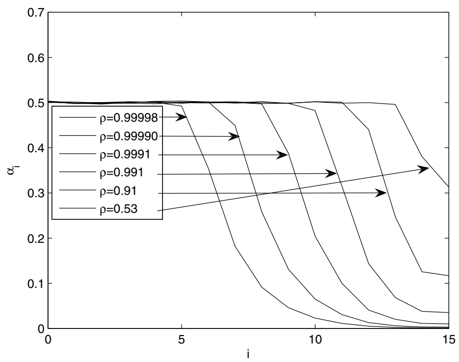

Fig. 1. Bit-level switching activity α_i for data streams with different ρ.

III. Word-Level Switching Activity

Figure 1 shows the simulated bit-level switching activity for several different data streams, with different correlation coefficients ρ. Here $i = 0$ is the index of the least significant bit. We have performed the simulations in the SystemC design environment, [7], using the signal model (1). The original signal $d(n)$ is generated for $D = 0.0005$ and $\sigma_d^2 = 0.5$ for all curves. To this we add noise signals $w(n)$ with $\sigma_w^2 = 5 \cdot 10^{-6}$, $5 \cdot 10^{-5}$, $5 \cdot 10^{-4}$, 0.005, 0.05, and 0.5 resulting in $z(n)$ signals with different ρ used as input to the simulations. As the figure shows, the switching activity for each case can be divided into three regions as predicted by the dual bit type method; The uniform white noise region (UWN) ($0 \le i \le P_0$), where bits have random switching with $\alpha_i = 0.5$, the sign (S) region ($P_1 \le i < W$), with switching activity α_{MSB}, and an intermediate region (I) ($P_0 < i < P_1$) where switching activity decreases from α_{UWN} to α_{MSB}. [2] simplifies the switching activity in the intermediate region by saying that the switching activity in this region decreases linearly from α_{UWN} to α_{MSB}, which can be seen from Figure 1 to be a reasonable approximation. This approximation can be summarized by the following switching activity model:

$$\hat{\alpha}_i = \begin{cases} 0.5 & i \le P_0 \\ 0.5 + (\alpha_{MSB} - 0.5)\frac{i-P_0}{P_1-P_0} & P_0 < i < P_1 \\ \alpha_{MSB} & i \ge P_1 \end{cases} \quad (4)$$

Figure 2 a) illustrates how regions for a W-bit signal according to the above model are divided.

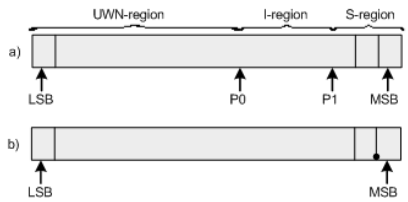

Fig. 2. Signal representation format. a) Integer format b) Decimal number format

Considering a W-bit signal $z(n)$, represented as $[z_0...z_{W-2}z_{W-1}]$, where z_{W-1} and z_0 are the most and least significant bits, respectively, the two's complement representation of $z(n)$ is given by:

$$z(n) = -z_{W-1}(n)2^{W-1} + \sum_{i=0}^{W-2} z_i(n)2^i \quad (5)$$

The value of $z(n)$ is an integer number in the range $-2^{W-1} \leq z(n) \leq 2^{W-1} - 1$. For the integer signal $z(n)$ in (5), [3] defines the region boundaries, P_0 and P_1 as:

$$P_0 = \text{ROUND}[\log_2(\sigma \cdot (1 - \rho_{MSB}))] \quad (6)$$

$$P_1 = \text{ROUND}[\log_2(6\sigma \cdot \sqrt{1 - \rho_{MSB}})] \quad (7)$$

where ROUND defines a rounding operation. ρ_{MSB} is the lag-1 correlation coefficient of z_{W-1}, and is defined by:

$$\begin{aligned} \rho_{z_i} &= \frac{E[z_i(n)z_i(n-1)] - E^2[z_i(n)]}{\sigma_{z_i}^2} \\ &= \frac{E[z_i(n)z_i(n-1)] - p_{z_i}^2}{p_{z_i} - p_{z_i}^2} \end{aligned} \quad (8)$$

where $p_{z_i} = \text{P}(z_i = 1) = E[z_i(n)]$, and $\sigma_{z_i}^2 = E[(z_i(n) - p_{z_i})^2] = p_{z_i} - p_{z_i}^2$ [8].

Real world signals are typically represented with an integer and a fractional part (W_I and W_F, respectively), separated by a binary point (.). Figure 2 b) illustrates such a format, with $W_I = 1$ and $W_F = 15$. In order to generalize the expressions (6) and (7) to this condition, we have extended the model in [3] by letting the W-bit signal $z(n)$, with W_I and W_F parts, be represented as $[z_0 \cdots z_{W_F-1} \, . \, z_{W-W_I} \cdots z_{W-2}z_{W-1}]$. Then the two's complement representation of $z(n)$ is given by:

$$z(n) = -z_{W-1}(n)2^{W_I-1} + \sum_{i=0}^{W-2} z_i(n)2^{i-W_F} \quad (9)$$

where $W = W_I + W_F$.

In order to use the above expressions to calculate the region boundaries, the signal described by (9) must be scaled so that the format for the signal becomes equivalent to the format of the signal described by (5). To do so the signal must be scaled by 2^{W_F}, where W_F is the number of bits to the left of the binary point ($z(n)$ is shifted W_F bits to the right.). The resulting expressions after such an adjustment become:

$$P_0 = \text{ROUND}[\log_2(\sigma \cdot 2^{W_F} \cdot (1 - \rho_{MSB}))] \quad (10)$$

$$P_1 = \text{ROUND}[\log_2(6\sigma \cdot 2^{W_F} \cdot \sqrt{1 - \rho_{MSB}})] \quad (11)$$

IV. Accurate estimation of α_{MSB}, ρ_{MSB}

It can be seen that accurate estimation of bit-level switching activity, α_i, using (4), (10), and (11) requires accurate estimation of ρ_{MSB} and α_{MSB} (ρ_i and α_i when $i = W - 1$). However, the present methods to calculate α_{MSB} and ρ_{MSB} ([3], [4]) are either based on the assumption that ρ_{MSB} is equal to the word-level correlation, which as mentioned earlier is not accurate in many cases, or are based on knowledge about the signal generation model in use. In [5] an accurate analytical expression for calculating α_{MSB} based on the word-level correlation is given by:

$$\alpha_{MSB} = \frac{1}{\pi} cos^{-1}(\rho) \quad (12)$$

It is also shown in [5] that the probability of a given bit i having the value 1 in a two's complement representation of a zero-mean Gaussian signal is $p_i = 0.5$. Further, in [4] an expression for the bit-level switching activity is given as:

$$\alpha_i = 2p_i(1 - p_i)(1 - \rho_i) \quad (13)$$

Now, as $p_i = 0.5$, α_{MSB} in (13) becomes:

$$\alpha_{MSB} = 0.5(1 - \rho_{MSB}) \quad (14)$$

By substituting α_{MSB} in (14) with (12), we can hence find an expression to calculate ρ_{MSB} directly:

$$\rho_{MSB} = \frac{2}{\pi} \sin^{-1}(\rho) \quad (15)$$

Figures 3 and 4 show the simulated results along with calculated results for α_{MSB} and ρ_{MSB}, respectively. For simulations the signal model (1) is used, with $D = 0.0005$, $\sigma_d^2 = 0.5$, and different values for σ_w^2. Simulations are run in the SystemC environment [7]. The results show a close match between simulated and calculated results. α_i and ρ_i, for the MSB, are simulated based on (2) and (8), respectively.

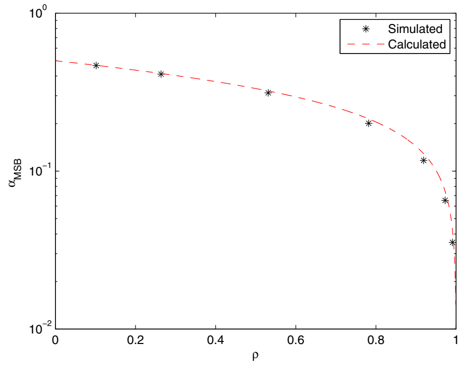

Fig. 3. α_{MSB} as function of ρ, for the simulated case along with α_{MSB} using (12).

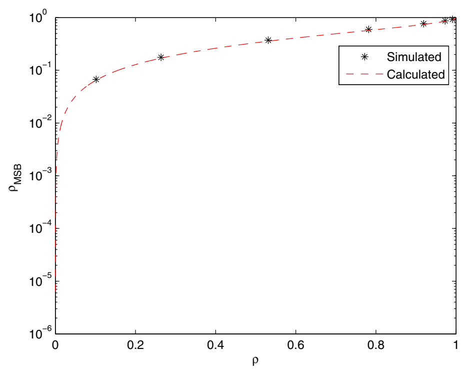

Fig. 4. ρ_{MSB} as function of ρ, for the simulated case along with ρ_{MSB} using (15).

V. Switching Activity of a Correlated Signal with Additive White Noise

Signals in a communication system are often subjected to noise. For example, fixed point realizations of DSPs, i.e., reduction in the level of precision of the DSP operations, introduce additional additive noise to the signal being operated on. In [9], it is shown how a small increase in this noise will decrease the performance of the channel estimation in a wireless receiver, leading to a considerable increase in the required estimator complexity if a given performance for the channel estimation must be upheld. In that regard, for the designer to be able to see how such additional noise will impact the power consumed by the corresponding DSP module, it is also important to analyze how such additional noise impacts the switching activity in the input signal. Another example of how noise can be added is through signal transmission over a suboptimal channel, wired or wireless.

To accurately estimate the switching activity in the input signal $z(n) = d(n) + w(n)$, we thus need to calculate the ρ_{MSB} and α_{MSB} for the signal $z(n)$. Here $w(n)$ is the noise signal being added to the signal $d(n)$, for instance due to a fixed-point DSP realization. The variance of $w(n)$ will depend on the number of bits in the realization [9].

In the following we will first present the expression for calculating ρ for the correlated signal $d(n)$, described in Section II, and then derive expressions for accurately calculating σ^2 and ρ when noise $w(n)$ is added to $d(n)$. We end this section by presenting simulation results, showing how the noise $w(n)$ added to the signal $d(n)$ impacts the switching activity in the signal.

The lag-1 autocorrelation of $d(n)$ normalized w.r. to σ^2 using (3), is given as:

$$\rho_d = \frac{R_d(1)}{\sigma^2} = J_0(2\pi D)$$

Since ρ_d is now available, $\rho_{d_{MSB}}$ and $\alpha_{d_{MSB}}$ can be calculated using (12) and (15) respectively. From this, the region boundaries in the input signal can be calculated. To calculate the same information for the noisy faded signal $z(n)$, we need to calculate σ_z and ρ_z when the signal $w(n)$ with a given variance is added to the signal $d(n)$. Assuming $d(n)$ and $w(n)$ to be independent, we have

$$\sigma_z^2 = \sigma_d^2 + \sigma_w^2,$$

and

$$R_z(k) = R_d(k) + R_w(k).$$

When $w(n)$ is white noise, we have $R_w(k) = \sigma_w^2 \delta[k]$, and we can easily find that

$$\rho_z = \frac{\rho_d \sigma_d^2}{\sigma_d^2 + \sigma_w^2}. \qquad (16)$$

Again we want to evaluate the accuracy of the derived analytical expression through a comparison with simulations using our signal model (1). Figure 5 shows a good match between simulated ρ_z, and ρ_z calculated using (16). The results are shown for different signals with Doppler rates of $D = 0.0005$, $D = 0.005$, $D = 0.05$, and $D = 0.2$, with variance of the noise added to the signal $d(n)$ being varied in each case. The variance of $d(n)$ is $\sigma_d^2 = 0.5$ in all cases.

Figure 6 shows the switching activity as function of bit position, for a fading signal of Doppler rate D=0.0005 as σ_w^2 added to the signal is varied. Results are shown both for the analytical expression derived in this section and from simulations. The curves demonstrate the accuracy of the used method to define the region boundaries. As we can see the breakpoints (i.e., the region boundaries) of the calculated and simulated curves show good agreement.

Figure 7 compares the word-level switching activity using the calculated method with simulated results.

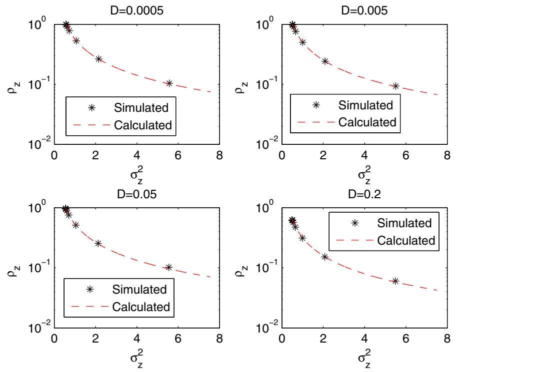

Fig. 5. Simulated ρ_z, along with ρ_z calculated using (16) as function of σ_z^2.

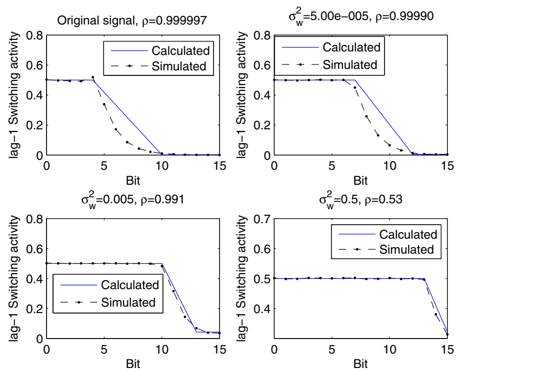

Fig. 6. Switching activity as function of bit position when σ_w^2 of different values is added to the original signal with D=0.0005. 'Sim' and Cal.' stand for the simulated and calculated case, respectively. For the calculated case α_i for the regions W_{UWN}, W_S, and N_I are estimated using (12) and (4), and the region boundaries are calculated using (10) and (11). Word length, $W = 16$ and $W_I = 3$.

Word-level switching activity for the simulated results is the sum of the simulated switching activity for all the bits in the word, and for the calculated method it is the sum of switching activity for all the bits in the word using (4) and (12). The figure shows good agreement between the curves for most ρ values. When ρ is very high (above 0.999) the difference in word-level switching activity increases. This is because the intermediate (I) region in these cases is wide, and the switching activity in this region does not decrease linearly from α_{UNW} to α_{MSB}, as assumed in (4). Thus the deviation is larger, as can be seen from the curves in Figure 6.

Figure 7 also shows that the word-level switching activity increases significantly when ρ decreases. A decrease in ρ corresponds to an increase in noise variance σ_w^2 (16). With decreasing ρ equal to 0.99998, 0.99990, 0.9990, 0.991, 0.91, 0.78, and 0.53, the corresponding increasing noise variances are $5 \cdot 10^{-6}$, $5 \cdot 10^{-5}$, $5 \cdot 10^{-4}$,

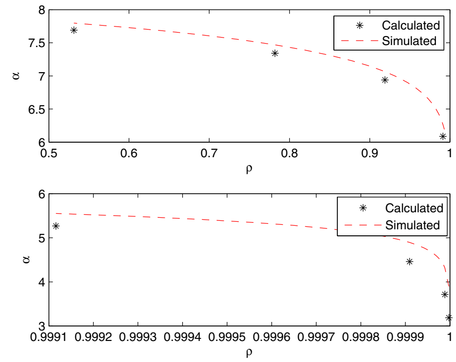

Fig. 7. Simulated word-level switching activity along with calculated word-level switching activity, for different ρ.

0.005, 0.05, 0.158, and 0.5. As the figure shows, for an added noise variance of $\sigma_w^2 = 0.005$, the word-level switching activity is increased to twice its initial value. The signal model used for simulations in Figures 6 and 7 is the model described in Section IV.

VI. CONCLUSIONS

In order to design circuits for low power, accurate estimation of power at different design stages has become important. As dynamic power consumption in CMOS VLSI circuits depends on the amount of signal transitions at the capacitive nodes, accurate estimation of the bit-level switching activity at processing units' inputs is a key requirement in various power estimation techniques. The DBT method aims at characterizing the bit-level switching activity in a signal, using signal statistics. However, the method requires that the correlation coefficient and switching activity for the most significant bit of the signal (α_{MSB} and ρ_{MSB}, respectively) are available. In this paper we have given an expression for direct calculation of ρ_{MSB} using ρ, based on a model for calculation of α_{MSB} presented in [5]. Comparing analytical results against simulation results we show that the given method to calculate α_{MSB} and ρ_{MSB} is highly accurate. Furthermore, we derive expressions for accurately calculating σ^2 and ρ for a correlated signal, when an additional noise of a given variance is added to the signal. These expressions are used to estimate bit-level switching activity in a signal, in the presence of noise of a given variance. The switching activity of a signal at the word-level is then the sum of the activities of all the individual bits constituting the signal. Using this method, and by adding noise of different variance to a correlated signal, we show the significant impact that additional noise can have on the switching activity in the signal. These results make it possible for the designer to model the actual input switching activity in different real life noisy environments, enabling realistic power consumption estimation.

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